OPTIMIZING THE NUMBER OF STOPS WITHIN THE LOGISTIC ROUTES USING THE MATHEMATICAL MODEL

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This paper is output solutions of scientific project VEGA 1/0380/17 – „Economic efficiency of electromobility in logistics".

Abstract: Presented paper focuses on the possibility of using mathematical models in marketing, specifically to optimize the number of stops on the distribution routes. It is devoted to proposing an allowance routes and number of stops on each route, length distribution routes and distances between stops on the route. Subsequently, the proposed procedures for the design of experiments as well as the statistical procedures and methods for processing measured data and the ways in which it is possible to evaluate the suitability of the data. There is designed a mathematical model through analysis of variance and other methods that describes the impacts on the logistics process. The conclusion is an evaluation of the use of the proposed model in practice, with specific methods to be evaluated. The most important of these is the method of response surface (whose output is a 3D graphical graph) through which we follow the characteristics of selected parameters which have the greatest impact on the process.

Keywords: *logistic, optimizing, mathematic model, distribution and logistics trends, innovation*

1. Introduction

Today, it is necessary to optimize operations and processes in order to increase their efficiency. The same applies in the area of logistics, or in designing and dealing with logistics routes. In analyzing the conditions and the subsequent design of logistics (distribution) route it is necessary to look at these activities from two perspectives. The first one is the time resources which are available at the said activities. In practice, this means that our efforts is to optimize distribution routes so that at that time the greatest number of stops is realized in order to serve the greatest number of customers (in this case to serve means to deliver goods). The other very important aspect is the financial aspect which is as important as the time required. In financial terms it is necessary to optimize distribution routes so that the costs necessary for the overall distribution routes are reduced to the minimum possible value. To realize such an optimization, it is necessary to know which factors affect the logistics process and upon influencing of which of them we can achieve such result. Precisely because of the knowledge of influential factors and their consequent influence, the next step is to create a model that describes the process. Finally, it is important to use the results of modeling and consequently of numerical or graphical interpretation. In graphical interpretation we use in this case a response surface, since it represents appropriate 3D display of correlation of influencing factors.

2. Impact specifications on logistics route

For each model creation it is important to firstly define the factors that affect the process. Subsequently, from such factors it is necessary to determine those ones whose impact is the most significant. Often, in the case of more complex processes there is the fact that in designing model not only the individual factors are used but also their mutual interactions, or higher-level factors. In the first step, therefore, we define the factors that affect logistics process, in this case affecting the logistics route and we briefly describe them:

Distance - is an essential factor from which further depends number of stops and the total length of the distribution route

Number of stops - this factor is based on the fundamental nature of the business and the related logistics. The aim is that the number of stops to a route is as high as possible, i.e. so that we are able to serve as many customers as possible

Time for one stop - represents the time required for the driver to unload or load goods at the customer. For our needs and better treatment of the issue, the one stop will represent time from the moment of stop at the client, unloading/loading of goods up to the next stop.

Total time on the route - this factor means the time spent by a driver on logistic route from the beginning till the moment of final parking of the vehicle.

Minimum time for one stop - is the minimum time which is necessary for stop and unloading and loading of the smallest contract possible, under ideal conditions

Financial costs of one stop - represent the sum of all costs calculated for the unit price for one stop expressed in €.

Costs of the entire route - this factor is very important as it represents the total costs of the logistics route. In practice, the sum of all logistic routes represents total costs of delivery of goods. The total costs is treated because the price of one stop reflects all costs associated with logistics calculated per one stop (salary of the driver, vehicle depreciation, fuel, etc.).

2.1 Selection of suitable factors

In the previous section we have defined all factors influencing the process which in our case is a selection of distribution routes. Further we choose a factor affecting the process in the largest way and with whom we will continue to work. The reason why such a selection of factors is made is that we don't want to work unnecessarily with too many factors whose impact on the process or on value measured by us is negligible. In our case, we chose the following factors: the Length of the route, Number of stops, One stop financial costs. These three variables are called the independent or explanatory variables. It's because these are variables that affect the final measured value. This value will be called dependent, since its variation depends on the change of the independent, explanatory, variables. In our case, the dependent variable value is: The costs of the entire route. The reason why we chose this value for our work is, as we have already described in the introduction, that we decided to optimize distribution route and therefore its stops as well with respect to its financial aspects, i.e. costs of such distribution route. In the next section we will show experimental design method through which we will measure all the necessary values of the factors so that we are able to create a model through them and consequently, in the graphic display, also response surface.

3. Creating the model and interpretation of results

Given the complexity of this process and the fact that we are working with three independent factors we use in our case non-linear model, the shape of which is:

$$w_i = a_0 + \sum_{j=1}^{k} a_j t_{ji} + \sum_{j=1}^{k-1} \sum_{l=j+1}^{k} a_{jl} t_{ij} t_{il} + \ldots + \sum_{j=1}^{k} a_{jj} t_j^2$$

(1.1)

where:

w_i represents ith measured dependent variable , a_0, a_u .... a_{kk} are unknown parameters,, t_{1i}, t_{2i}, ..., t_{ji} are individual parameters and their interactions

In our case, due to the high number of examined factors and number of their levels (the level of individual factors will be chosen with regard to their feasibility and credibility in the home process) we have decided for centred composite design of experiment consisting of three parts:

<u>Core of design</u> – is formed by factorial design 2^k. As it will be shown below, if we want to reduce the number of measurements, the core can be used as reduced experiment 2^{k-p}. For number of

factors $k<4$ only complete experiment 2^k can be used as a core, for $5 < k < 7$ we use the complete experiment 2^k or the reduced experiment 2^{k-p} as a core, for $k > 7$ also reduced experiment 2^{k-2},

<u>*Axial points*</u> – points that are on the axes in distance of $a > 0$ from the centre of design. Their number is $n_s = 2k$,

<u>*Central points*</u> – points in the centre of the design. There are $n_0 > 0$ of central points. By suitable selection of "*a*" and the number of central points - "$n_{0,}$" the proposals characteristics can be influenced. Centred composite design is presented in Table. 1.0.

Tab. 1: Centred composite design

t_1	t_2	...	t_k	Number of points	Design part
-1 -1 ... +1	-i -i ... +i	... ...	-1 +1 ... +1	$n_c=2^k$ or $n_c = 2^{k-p}$	Design core
-α +α 0 0 ... 0 0	0 0 -a +a ... 0 0	... ...	0 0 0 0 ... -α +α	$n_s=2k$	Axial points
0 0 ... 0	0 0 ... 0		0 0 ... 0	n_0	Central points

Source: SVÁTEK, M. - KRIŽAN, P. 2015

In the next step, the individual measurements and the data processing or measured data analysis proceeds. During the data processing, firstly the data are verified by the following tests:

<u>Normal distribution verification</u>: *Shapiro-Wilk test*

<u>Diagnosing the residuals</u>: S*tudentized residuals*

<u>Outliers determination</u>: *Dean-Dixon* non-parametric test of outliers and a *Grubbs test*

<u>Testing of homoscedasticity of dispersions:</u> *Levene test* or its modification *Brown-Forsythe test*, or *Bartlett* test. Its disadvantage is the high sensitivity against normality violation.

Then, after the data analysis, we proceed with the experiment model design using the ANOVA method (all of the above analysis, because of the complexity of matrix notation, effected through the available statistical software, for example JMP, STATGRAPHICS etc.):

In our case it is the experiment *a* x *b* x *c*, because we consider 3 factors while factor A has *a* levels, B has *b* and *C* has *c* levels, etc.

The model of complete factorial design which contains all possible interactions is in our case of 3-factorial design as follows:

$$y_{ijklm} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijklm}$$

(1.2)

where

μ is the rate constant, or also called absolute term,

α_i is the contribution of i-times *level of* factor *A*,

β_j is the contribution of j-times level of factor *B*,

γ_k is the contribution of k-times level of factor *C*,

$(\alpha\beta)_{ij}$ is combined contribution of i-times level of factor A and j-times level of factor B (interaction)

$(\alpha\gamma)_{ik}$, $(\beta\gamma)_{jk}$, $(\alpha\beta\gamma)_{ijk}$, …, etc., individual combined contributions of given factors (their mutual interactions),

ε_{ijklm} is contribution of m-times observation (error)

while $i, j, k, l, = 1, 2, ... , m$

Subsequently, from the thus calculated values a 3D graph will be created, the area of which represents a characteristic behavior of the said process and is called response surface.

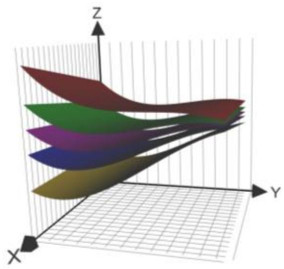
4. Conclusion

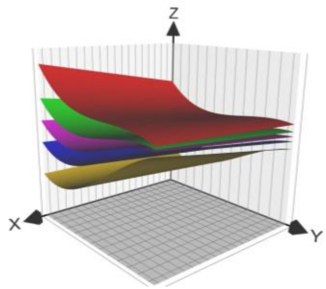
In the interpretation and subsequent application of the result a numerical or graphical form of the result may be used. In the first case it is for clarity of our dependent variable, depending on the change of the independent variables, or vice versa. In the second case, for example, the response surface is used which represents a 3D graph whilst surface or its curvature expresses dependence or change in the dependent variable if two independent variables change.

Fig. 1 illustrates a sample of response surface graph and this graph may be used for cases of analysis of dependency of the change in total costs (z axis) on change of number of stops (x axis) and the costs of one stop (y axis), or to express the dependence of the length of route (z axis) on the costs claimed (x axis) and the total number of stops along the route (y axis). In general it can be said that the response surface is a very good tool for the graphical representation of the changes of the observed variable depending on changes in the other two variables whose influence on it we follow.

In our case, as we have said, we have focused on optimizing distribution route, namely the number of stops in regard to its financial (costs) aspect. As we mentioned in the same way, by defining the appropriate factors an experiment and subsequent evaluation of distribution routes may be realized, or optimize it with regard to the time available to us.

Fig. 1: Surface response with several level settings





Source: Križan, P. - Šooš, Ľ. - Matúš, M. - Beniak, J. - Svátek, M.: 2015

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Primary Paper Section: A

Secondary Paper Section: AH, BA